1.1) 

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1.2) (DDD, ddd) is the only N.E. in RNF.

1.3) (DDD, x_1, ddx, ddx, ddx, x_2, x_1), x_1, x_2 e E(C, D), are all N.E. in full N.E., (0,0) is the outcome for all N.E. Only (DDDDDD, dddddd) is SPE.

1.4) NLOG, say player 1 plays C on the path:

Time T: Since it is the last period, and since D strictly dominates C, Player 1 will deviate. Thus $S_t(h_{t-1}) = D$.

Time T-1: Since next period he will play D, there is no punishment for deviation, he will deviate: $S_t(h_{t-2}) = D$.

Time 1: Since for all subsequent periods, player 1 will play D, there is again no punishment for deviation. Thus he would deviate. Thus $S_t(h_{t-1}) = D$.
1.9 cont'd) At all periods, if Player 2 plays C there will be
incentive to deviate. Thus, since the opposing player's
strategy was not a factor, Player 2 will play D always.
Thus no player can play C on the equilibrium path.

2) See HW #3 AK

3.1) Player 1's acceptance rules: accept if:
\[ x_1^2 \geq S_1 x_1^1 \quad \text{and} \quad x_1^3 \geq S_1 x_1^1 \]

Player 2's acceptance rules: accept if:
\[ x_2^3 \geq S_2 x_2^1 \quad \text{and} \quad x_2^3 \geq S_2 x_2^1 \]

Player 3's acceptance rules: accept if:
\[ x_3^1 \geq S_3 x_3^1 \quad \text{and} \quad x_3^2 \geq S_3 x_3^2 \]

Feasibility:
\[ x_1^1 + x_2^1 + x_3^1 = 1 \quad \text{and} \quad x_1^2 + x_2^2 + x_3^2 = 1 \quad \text{and} \quad x_1^3 + x_2^3 + x_3^3 = 1 \]

3.2) See program Pr3-2.

3.3) If we let \( S_1 = S_2 = S_3 = 1 \), then we get:
\[ x_1^2 = x_1^3 = x_1^1 = \alpha \quad \text{and} \quad x_2^3 = x_2^1 = x_2^2 = \beta \quad \text{and} \quad x_3^1 = x_3^2 = x_3^3 = \gamma \]
\[ x^1 = x^2 = x^3 = (\alpha, \beta, \gamma) \]. Thus, for acceptance rules:
Player 1 accepts if: Player 2 accepts if: Player 3 accepts if:
\[ x_1^2 \geq \alpha, \quad x_1^3 \geq \alpha, \quad x_2^2 \geq \beta, \quad x_2^3 \geq \beta, \quad x_3^2 \geq \gamma, \quad x_3^3 \geq \gamma \]
3.3 (contd) and offers as listed above, then any split $(a, b, c)$ can be supported if $o = 1$.

4.1) \[ C \quad d_a \quad d_b \quad \text{pure NE: } (d_A, d_A) \quad (d_B, d_B) \]
\begin{align*}
C & \quad b & \quad 2,2 & \quad 2,1 \\
D_A & \quad 2 & \quad 2 & \quad -1,-1 \\
D_B & \quad 2 & \quad -1,-1 & \quad 0,0 \\
\end{align*}

4.2) \[ S_1(h_o) = C , \quad S_2(h_o) = c \]
\[ S_1(h_i) = \begin{cases} D_A & \text{if } h_i = Cc \\ D_B & \text{else} \end{cases} , \quad S_2(h_i) = \begin{cases} d_A & \text{if } h_i = Cc \\ d_B & \text{else} \end{cases} \]
\[ U^i(s^*) = 3 + 2 = 5 \]

Since the game and strategies are symmetric, we only have to check one player's deviations:

Player 1: at time 2:
- given any history ($h_i = Cc$, or $h_i \neq Cc$), we are already best responding, thus no profitable deviations

at time 1:
- best deviation is $D_A \Rightarrow U^i(s) = 4 + 0 = 4 < U(s^*)$

Thus Player 1 has no profitable deviations.

At all times, following all histories, neither player has a profitable deviation, thus this is SPE.
(4.3) \( S^*_1(h_0) = DA \quad S^*_2(h_0) = c \)

\[
S^*_t(h_t) = \begin{cases} 
DA & \text{if } h_t = DAC \\
DB & \text{else}
\end{cases}
\]

\[
S^*_t(h_t) = \begin{cases} 
da & \text{if } h_t = DAC \\
db & \text{else}
\end{cases}
\]

- Again, in stages 2-4, neither player will deviate following any history since \( DA \in eB(da) \), \( DB \in eB(db) \), \( da \in eB(Da) \), \( db \in eB(DB) \).

- Player 1 won’t deviate in time 1 since \( DA \in eB(c) \).

- Player 2’s best deviation in time 1 is \( da \):

\[
U^2(S^*_1) = 2 + 0 + 0 + 0 < 2 + 2 + 2 + 2 = U^2(S^*_0)
\]

Thus at all times, following all histories there are no profitable deviations. Thus \( S^* \) is SRE.

(4.4) Let \( h^*_1 = (DA, db) \), \( h^*_2 = \{DA, db, (DB, da)\} \), \( h^*_t = h^*_{t-1}, (DA, da) \) \( t \geq 3. \)

Then \( S^*_1(h_0) = DA \), \( S^*_2(h_0) = DB \)

\[
S^*_t(h_t) = \begin{cases} 
\text{follow path} & \text{if } h_{t-1} = h^*_{t-1} \\
DB & \text{else}
\end{cases}
\]

\[
S^*_t(h_t) = \begin{cases} 
da & \text{if } h_{t-1} = h^*_{t-1} \\
db & \text{else}
\end{cases}
\]

for \( 2 \leq t \leq T. \)

Then \( U^*(S^*) = -1 - 1 + 2(T-2) = 2T - 6. \)

- No player will deviate in \( t \geq 3 \) rounds, since \( DA \in eB(da) \), \( DA \in eB(da) \), \( DB \in eB(DB) \), and \( da \in eB(DA) \).
4.9 cont'd)

round 1: player 1 deviates: \( DB \in \beta(d) \)

\[ \Rightarrow u'(s) = 0 \leq 2T-b \text{ when } T \geq 3 \]

player 2 deviates: \( DA \in \beta(D) \)

\[ \Rightarrow u^2(s) = 2 \leq 2T-b \text{ when } T \geq 4 \]

round 2: player 1 deviates: \( DA \in \beta(dA) \)

\[ \Rightarrow u'(s) = -1 + 2 = 1 \leq 2T-b \text{ when } T \geq 3.5 \]

player 2 deviates: \( DB \in \beta(DB) \)

\[ \Rightarrow u^2(s) = -1 \leq 2T-b \text{ when } T \geq 2.5 \]

Thus, \( T = 4 \) is sufficient for this path to be SPE.

5.1) True. proof:

Let \( s^* \) be a history independent SPE, and let, in period 1, \( s^* \) induce non-stage game NE play. Thus, in period 1, there is at least one player who has a deviation that results in a strictly better payoff in that stage game. Since \( s^* \) is history independent, continuation play is unaffected by this deviation and thus at least one player can profitably deviate from \( s^* \) in \( G^1 \). This \( G^1 \) is not SPE. Therefore, it must be that stage-game NE play is induced in every period under \( s^* \).
5.2) True. Proof:

Obvious. There are no profitable deviations, since all $s^*$ are history independent, resulting in stage-game N.E. play. Considering play after all histories is trivial in this case.

5.4) false: counterexample:

Consider a $T$-repeated game based on the stage game from problem 4. The following strategy induces stage-game Nosh play in all periods:

\[ s_1^*(h_0) = D_B \quad s_1^*(h_0) = D_B \]

\[ s_2^*(h_{t-1}) = \begin{cases} D_B & \text{if } h_{t-1} \in \{D_B,d_B\}, \{D_B,d_B\}, \ldots, 3 \\ D_A & \text{else} \end{cases} \]

\[ s_2^*(h_{t-1}) = \begin{cases} D_B & \text{if } h_{t-1} \in \{D_B,d_B\}, \{D_B,d_B\}, \ldots, 3 \\ d_B & \text{else} \end{cases} \]

It is clear, though, that deviations are profitable in all periods but the last. Thus it is not S.P.E.