Assignment 6

1. Consider the normal form game

<table>
<thead>
<tr>
<th></th>
<th>Straight</th>
<th>Swerve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight</td>
<td>0,0</td>
<td>4,1</td>
</tr>
<tr>
<td>Swerve</td>
<td>1,4</td>
<td>3,3</td>
</tr>
</tbody>
</table>

1. Solve for all equilibria.
2. Consider a sequence of perturbations \( \{G_k\}_{k=1}^{\infty} \) where for each \( G_k \) player \( i \) has type \( \theta_i \) which is uniform on \([0, \frac{1}{k}]\) and where the payoffs are

<table>
<thead>
<tr>
<th></th>
<th>Straight</th>
<th>Swerve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight</td>
<td>0,0</td>
<td>4+\theta_1,1</td>
</tr>
<tr>
<td>Swerve</td>
<td>1.4+\theta_2</td>
<td>3,3</td>
</tr>
</tbody>
</table>

Find all pure strategy equilibria in the incomplete information game and demonstrate that the observed frequencies converge towards the mixed strategy equilibrium in the original game as \( k \to \infty \).

2. Consider a first price auction with three symmetric bidders in which valuations are independently and uniformly drawn from interval \([3, 9]\). Find a symmetric equilibrium in strictly increasing strategies.

3. Consider the following static incomplete information game. With probability \( \alpha \) player 1 is "strong" in which case the payoffs are given by the matrix

\[
F = \begin{pmatrix}
2 & 0 \\
3 & 1 \\
\end{pmatrix}
\]

and with probability \( (1 - \alpha) \) player 1 is “weak” in which case the payoffs are

\[
F = \begin{pmatrix}
0 & 0 \\
3 & 1 \\
\end{pmatrix}
\]

Denote the type space by \( \Theta = \{s, w\} \)

1. Define the strategy sets for the two players (note that play is simultaneous).
2. Write down the Normal form payoffs as a standard matrix form game.
3. For \( \alpha = \frac{1}{2} \) find all pure strategy Nash equilibria.
4. Consider a “war of attrition”, which a two player game where an action is a number \( a_i \in A = [0, \infty) \), which can be interpreted as when the player will drop out. Types are i.i.d on \( \Theta = [0, \infty) \) with cdf \( F \) and pdf \( f \) and the payoffs are

\[
u_1(a_i, a_j, \theta_i) = \begin{cases}
-a_i & \text{if } a_i \leq a_j \\
\theta_i - a_j & \text{if } a_i > a_j
\end{cases}
\]

1. Suppose that \( s^* \) is an equilibrium. Write down the appropriate best response conditions that must hold.
2. (Hard! Skip if you get stuck) Show that any equilibrium must be strictly increasing.
3. Using the guess that there is a symmetric equilibrium in strictly increasing strategies, formulate the appropriate optimization problem.
4. Derive a general expression for \( s^*(\theta) \) in a symmetric equilibrium.