Assignment 7

1. Consider the following static incomplete information game. With probability $\alpha$ player 1 is "strong" in which case the payoffs are given by the matrix

$$
\begin{pmatrix}
F & D \\
F_s & 2, 0 \\
D_s & 1, 2 \\
\end{pmatrix}
$$

and with probability $(1 - \alpha)$ player 1 is “weak” in which case the payoffs are

$$
\begin{pmatrix}
F & D \\
F_w & 0, 0 \\
D_w & 1, 2 \\
\end{pmatrix}
$$

Denote the type space by $\Theta = \{s, w\}$

1. Define the strategy sets for the two players (note that play is simultaneous).
2. Write down the Normal form payoffs as a standard matrix form game.
3. For $\alpha = \frac{1}{2}$ find all pure strategy Nash equilibria.

2. Consider 2 players playing the a game with incomplete information where player 1 is privately informed about $\theta$ which is uniformly distributed on $[0, 1]$ and where the payoffs are

$$
\begin{pmatrix}
L & R \\
U & 1 - \theta & 1 - \theta & 0, 0 \\
D & b, 0 & \theta + b, \theta \\
\end{pmatrix}
$$

where $b \in (0, 1)$.

1. Find all equilibria in the static game of incomplete information where moves are simultaneous.
2. Add a communication stage where player 1 sends a costless message $m \in \{m_1, m_2\}$ before any other actions are taken. Under appropriate conditions on $b$ find an equilibrium in which player 2 plays $L$ if $m = m_1$ and $R$ if $m = m_2$ and state the condition on $b$ for this to be consistent with equilibrium.

3. Consider a first price auction with three symmetric bidders in which valuations are independently and uniformly drawn from interval $[1, 6]$. Find a symmetric equilibrium in strictly increasing strategies.

4. Consider the following dynamic game of incomplete information. At time 0 “nature” draws the type of player 1 (the sender), which may be “strong” or “weak” where $p \in (0, 1)$ is the probability that the sender is strong. Player 2, the receiver, likes to fight with the sender if he is weak, but doesn’t want to fight if he is strong. However, before 2 is making the decision of whether or not to pick a fight, player 1 can have one of two possible breakfasts: either he starts his day with a pitcher of beer, or he may have a quiche. The “strong” type likes beer and the “weak” type likes quiche. However, both would like to avoid a fight. Concretely, suppose that payoffs are additive and that;

For the weak type:

$$
\begin{pmatrix}
\text{Beer} & \text{Quiche} \\
\text{Fight} & 0 & 1 \\
\text{Don’t} & 2 & 3 \\
\end{pmatrix}
$$
For the strong type

<table>
<thead>
<tr>
<th></th>
<th>Beer</th>
<th>Quiche</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fight</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Don’t</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Finally, the receiver gets the following payoffs

<table>
<thead>
<tr>
<th>Sender Weak</th>
<th>Sender Strong</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fight</td>
<td>1</td>
</tr>
<tr>
<td>Don’t</td>
<td>0</td>
</tr>
</tbody>
</table>

1. Use the information above to draw the associated extensive form of the signalling game described.
2. Under what conditions on $p$ is there a pooling equilibrium where both players have beer for breakfast?
3. Under what conditions on $p$ is there a pooling equilibrium where both players have quiche for breakfast?
4. Under what conditions on $p$ is there a separating equilibrium?
5. Under what conditions on $p$ is there a semi-separating equilibrium?

5. Consider the following simplification of poker with only two players.

First, each player first puts a dollar down (the ante). Notice however that there is no choice here, so you don’t need to make this part of the extensive form of the game (the reason for describing the setup this way is that it is a natural way of generating the payoffs).

Then player 1 draws a single card from a deck consisting of an equal number of queens and kings. Only player 1 draws a card.

After observing the card, player 1 may either “bet” or “fold”. If player 1 folds, player 2 wins the pot containing the ante (implying that player 2 wins 1 and player 1 loses 1).

If player 1 “bets”, he/she places another dollar in the pot. Player 2 then has the option between “fold” and “call”. If folding, player 1 takes the pot (implying a net transfer of 1 from player 2 to player 1). If player 2 “calls” he/she adds another dollar to the pot. In this case player 1 wins if he has a king (in which case there is a net transfer of 2 player 2 to player 1) and player 2 wins if the professor has a queen (in which case there is a net transfer of 2 player 1 to player 2).

1. Draw the extensive form of the game.
2. Derive the associated reduced normal form.
3. Find all Nash equilibria in pure as well as mixed strategies.
4. Is the game “fair” in the sense that both players can expect to break even on average?