2.1)

<table>
<thead>
<tr>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>1-θ, 1-θ</td>
</tr>
<tr>
<td>D</td>
<td>b,0</td>
</tr>
</tbody>
</table>

Will player 2 mix? Let \( p = P(A_1 = U) \)

Then \( \pi(L) = \pi(R) \) implies:

\[
p(1-\theta) + (1-p)b = (1-p)(\theta+b) \iff p(1-\theta) = (1-p)\theta
\]

\( \iff p = \theta \). This is clearly not a condition we can make hold. Thus player 2 will not mix.

Player 2 plays \( R \).

Then \( S_1 = D \) is best response, and player 2 will not deviate, as \( E[\theta]g = \frac{1}{2} > 0 \)

Player 2 plays \( L \).

Then

\[
S_1 = \begin{cases} 
U & \text{if } \theta \leq 1-b \\
D & \text{if } \theta > 1-b
\end{cases}
\]

will player 2 deviate? \( \pi(L) = \pi(R) \iff

\[
\Rightarrow \int_0^{1-b} (1-\theta) d\theta \geq \int_{1-b}^{b} \theta d\theta \iff 1-b - \frac{(1-b)^2}{2} \geq \frac{1}{2} - \frac{(1-b)^2}{2}
\]

\( \iff b \leq \frac{1}{2} \). So if \( b \leq \frac{1}{2} \), \((S_1, L)\) is PNE.
2.2) Propose this strategy profile:

\[ S_1 = \begin{cases} m_1 U & \text{if } \Theta \leq \frac{1 - b}{2} \\ m_2 D & \text{if } \Theta > \frac{1 - b}{2} \end{cases} \quad S_2 = \begin{cases} L & \text{if } m = m_1 \\ R & \text{if } m = m_2 \end{cases} \]

Clearly, player 2 is best responding. Since player 1's strategy is separating in \( (\Theta_1 \Theta \leq \frac{1 - b}{2}), (\Theta_1 \Theta > \frac{1 - b}{2}) \), the analysis is easy.

Player 1 can deviate to any outcome in the game, given player 2's strategy. Thus his cutoff is designed to let him choose the best outcome. Because of this, there is no deviation that is profitable.

3) See HW #6.

For \( \Theta \in [0, 6] \) and \( N = 3 \), \( b_i(\Theta) = \frac{2}{3} \Theta_i + \frac{1}{3} \)

or \( b_i(\Theta) = \frac{2}{3} (\Theta_i - 1) + 1 \).

4.1)
4.2) Pooling in Bees: \( B_5 B_w \).

\[ m = \frac{p(B_5 | B_w) P(S)}{P(B_5) P(S) + P(B_1) P(w)} = \frac{p}{p + 1 - p} = p. \]

\[ S_2 | Q = \_? \]

If \( S_2 | Q = N, \) \( \Pi_1(Q | w) = 3 > \Pi_1(B_1 | w), \) regardless of \( S_2 | B, \)

Thus, in order to have \( S_1 | w = B, \) \( S_2 | Q = F \) must be.

\[ S_2 | B = \_? \]

If \( S_2 | B = F, \) \( \Pi_1(Q | w) = 1 > \Pi_1(B_1 | w), \) given \( S_2 | Q = F. \)

Thus, in order to have \( S_1 | w = B, \) \( S_2 | B = N \) must be.

Will player 2 deviate?

Since \( S_2 | Q = F, \) \( \Pi_2(F | Q) = 1 - \lambda \geq \lambda = \Pi_2(N | Q) \Rightarrow \lambda \leq \frac{1}{2} \)

Since \( S_2 | B = N, \) \( \Pi_2(N | B) = m \geq 1 - m = \Pi_2(F | B) \Rightarrow m = \frac{1}{2} \)

Will player 1 deviate?

No. We designed player 2's strategy so that 1 won't deviate. \( \Pi(B_5) = 3 > 0 = \Pi(Q | b) \) and \( \Pi(B_1 | w) = 2 > 1 = \Pi(Q | w). \)

Thus \( (B_5 B_w, N, F, Q) \) and \( m = p, \lambda \leq \frac{1}{2} \) is a PBE

for \( p = \frac{1}{2} \).
4.3) Pooling in Quiche: QsQw

\[ \lambda \] is generated using Bayes rule, \( \lambda = \rho \) (as previously shown)

\[ S_2|B = F \], since if \( S_2|B = N \), "strong" type would deviate regardless of \( S_2|Q \).

\[ S_2|Q = N \], since if \( S_2|Q = F \), gives \( S_2|B = F \), "strong" type would deviate to Beer.

Will player 2 deviate?

\[ \Pi_2(F|B) = 1 - M \geq M = \Pi_2(N|B) \Rightarrow \quad \lambda \leq \frac{1}{2} \]

\[ \Pi_2(N|Q) = 1 - 1 = \Pi_2(F|Q) \Rightarrow \quad \lambda \geq \frac{1}{2} \]

Will player 1 deviate?

Again, \( \Pi(Q|S) = 2 > 1 = \Pi(B|S) \) and \( \Pi(Q|W) = 2 > 0 = \Pi(B|w) \).

Thus \( (QsQw, FB\bar{N}Q) \) and \( \lambda = \rho, M \leq \frac{1}{2} \) is a PBE for \( p = \frac{1}{2} \).

4.4) \( S_1 = B_sQw \) \( \rightarrow \) by bayes rule, \( M = 1 - \lambda = 1 \)

and \( S_2 = N_bFQ \) \( \rightarrow \) now \( S_2|W = B \) is a profitable deviation for player 1. No PBE.

\( S_1 = Qs\bar{B}w \) \( \rightarrow \) by bayes rule, \( \lambda = 1 - M = 1 \)

and \( S_2 = FB\bar{N}Q \) \( \rightarrow \) now \( S_2|W = Q \) is a profitable deviation for player 1. No PBE.
4.5) Try: "Strong" mixes?

- Let $s_1 | w = B$, then $1 - 1 = 2$ $\Rightarrow$ $S_2 | Q = N$ and "Weak" will want to deviate, regardless of $S_2 | B$, since $\Pi_1(Q | w) = 3 > \Pi_1(B | w)$.

- Let $s_1 | w = Q$, then $1 - 1 = 2$ $\Rightarrow$ $S_2 | B = N$ and "Strong" cannot mix, regardless of $S_2 | Q$, since $\Pi_1(B | s) = 3 > \Pi_1(Q | s)$.

$\Rightarrow$ No PBE where "Strong" mixes

Try: "Weak" mixes?

- Let $s_1 | s = \bar{Q}$, then $1 - 1 = 2$ $\Rightarrow$ $S_2 | \bar{B} = F$ and "Weak" cannot mix, regardless of $S_2 | Q$, since $\Pi_1(B | w) = 0 < \Pi_1(Q | w)$.

- Let $s_1 | s = B$, then $1 - 1 = 2$ $\Rightarrow$ $S_2 | Q = F$.

- For "Weak" to mix, he must be indifferent.

Clearly, player 2 must mix following beer, let $r = P(\bar{Q} | \bar{B})$. Then the condition for mixing:

$\Pi_1(\bar{B} | w) = 2r - 1 = \Pi(Q | w)$ $\Rightarrow$ $r = \frac{1}{2}$

- For player 2 to mix following beer, he must be indifferent. We must calculate his beliefs:

$m = P(S | B) = \frac{P(B | s)P(s)}{P(B | s)P(s) + P(B | \bar{s})P(\bar{s})} = \frac{p}{p + q(1-p)}$ $\Rightarrow$ $q = P(B | \bar{s})$. 


4.5) cont'd: Condition for player 2 mixing:

\[ \pi(N|B) = m = 1 - m = \pi(F|B) \]

\[ \Rightarrow m = \frac{1}{2} \Rightarrow \frac{p}{p + 2(1-p)} \Rightarrow q = \frac{p}{1-p} \]

We can get bounds on \( p \) through this and \( q \in (0, 1) \).

\[ q > 0 \Rightarrow \boxed{p > 0} \quad \text{and} \quad q < 1 \Rightarrow \boxed{p < 1} \]

Thus: if \( p < \frac{1}{2} \):

\[ s_1 = \begin{cases} B & \text{if } \Theta = S \\ B & \text{if } \Theta = W \text{ w.p. } \frac{p}{p} \\ Q & \text{if } \Theta = W \text{ w.p. } \frac{1-2p}{1-p} \end{cases} \]

\[ s_2 = \begin{cases} F & \text{if } m = Q \\ F & \text{if } m = B \text{ w.p. } \frac{1}{2} \\ N & \text{if } m = B \text{ w.p. } \frac{1}{2} \end{cases} \]

and beliefs \( \lambda = 0 \), \( m = \frac{1}{2} \).

\( s, m^2 \) is PBE.

5.1)
5.2) Since player 2's moves following Fold are not played, we can simplify his strategy to a singleton response to a bet:

\[
\begin{array}{c|cc}
 & C & F \\
B_1B_2 & 0,0 & 1,1 \\
B_1F_2 & \frac{1}{2},\frac{1}{2} & 0,0 \\
F_1B_2 & -\frac{1}{2},\frac{1}{2} & 0,0 \\
F_1F_2 & -1,1 & -1,1 \\
\end{array}
\]

5.3) Clearly no pure strategy Nash. For player 1, folding after "king" is strictly dominated. So, let 
\[p = P(B1\vert Q), \quad 1-p = P(F1\vert Q), \quad 1 = P(B1\vert K), \quad 2 = P(C1\vert B)\]
and 
\[Q = P(F1\vert B)\]

**Player 1 mixing condition:**

\[\Pi_1(B1\vert Q) = -2(2) + (1-2) = -1 = \Pi(F1\vert Q) \iff Q = 2/3\]

**Player 2 mixing conditions:** First, beliefs:

\[M = P(1\vert B) = \frac{P(B1\vert K)P(K)}{P(B1\vert K)P(K) + P(B1\vert Q)P(Q)} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}p} = \frac{1}{1+p}\]

\[\Pi_2(C1\vert B) = M(-2) + (1-M)2 = -1 = \Pi(F1\vert B) \iff M = \frac{3}{4}\]

\[\iff 3Q = \frac{1}{1+p} \iff p = \frac{1}{3}\]
5.3) cont'd:

Thus, let

\[ S_1 = \begin{cases} 
  B & \text{if } \Theta = k \\
  S & \text{if } \Theta = q \quad \text{w.p. } -1/3 \\
  F & \text{if } \Theta = q \quad \text{w.p. } 1/3 
\end{cases} \]

\[ S_2 = \begin{cases} 
  C & \text{w.p. } 2/3 \\
  F & \text{w.p. } 1/3 
\end{cases} \]

\[ M = 3/4 \quad \text{then } S, M^3 \text{ is P.B.E.} \]

5.4) Not fair:

\[
E[\pi_2] = \frac{1}{2} \left( 2 \left( \frac{2}{3} \right) + \frac{1}{3} \right) + \frac{1}{2} \left( \frac{1}{3} \left( -2 \left( \frac{2}{3} \right) + \frac{1}{3} \right) + \frac{2}{3} \left( -1 \right) \right)
\]

\[
= \frac{5}{6} + \frac{1}{2} \left( -\frac{1}{3} - \frac{2}{3} \right) = \frac{2}{3} - \frac{1}{3}
\]

\[
E[\pi_2] = -E[\pi_1] = \left[ \frac{1}{3} \right]
\]