2.2) If $S^*_1 = S_{st} L_{wk}$, then $\delta = 0$ and $S^*_2 = D_L M_S$ and $u_1(S^* | \theta = wk) = 1 - \frac{1}{2} < u_1(Short, S^*_2 | \theta = wk) = 1$. For $\delta > 0$, this separating does not exist.

If $S^*_1 = L_{st} S_{wk}$, then $\delta = 0$, $\delta = 1$, $S^*_2 = M_L D_S$ Player 2 is already best responding.

"Strong" deviates?

$u(S^* | \theta = st) = 2\delta \geq 1 = u(S_{st}, S^*_1 | \theta = st) \iff \delta \geq \frac{1}{2}$

"Weak" deviates?

$u(S^* | \theta = wk) = 1 \cdot 2 \cdot (1 - \frac{1}{2})^2 = u(L_{wk}, S^*_1 | \theta = wk) \iff \delta \geq \frac{1}{2}$
2.2) Cont'd: no additional constraints are necessary on $p/2$.

\[ P(st|Alive) = \frac{P(Alive|st) \cdot P(st)}{P(Alive)} = \frac{2p}{2p + 1 - p}. \]

2.3) Pooling on Long? $s^*_1 = s_{st+1-wk}$.

Clearly, $s^*_2 = M \cap D_s$ is necessary, but is that enough to keep player 1 from deviating?

"Strong" deviates?

\[ n(s^*_1|\Theta = st) = 2 \quad \Rightarrow \quad n(s_{st}, s^*_2|\Theta = st) \quad \Rightarrow \quad q = \frac{1}{2} \]

"Weak" deviates?

\[ n(s^*_1|\Theta = wk) = 2(1-q) \quad \Rightarrow \quad n(s_{wk}, s^*_2|\Theta = wk) \quad \Rightarrow \quad 2q \leq \frac{1}{2} \]

\[ \Rightarrow q = \frac{1}{2} \quad \text{is necessary, but } q > \frac{1}{2}. \quad \text{Thus this pooling does not exist.} \]

Pooling on Short? $s^*_1 = s_{st+1-wk}$, $s^*_2 = D_s \cap M_s$

Player 2 deviates?

following long: $0 \geq x - (1 - x) \quad \Rightarrow \quad \boxed{x \leq \frac{1}{2}}$

following short: $m: P(st|s) = \frac{P(st|s_{st}) \cdot P(st)}{P(s)} = \frac{p}{p + 1 - p} = \frac{p}{2} - p$. \[ m - (1 - m) \geq 0 \quad \Rightarrow \quad m \geq \frac{1}{2} \]

Player 1 won't deviate.

So $(s^*_1, s^*_2, E^*_a)$ is PBE for $m = p = \frac{1}{2}$ and $x \leq \frac{1}{2}$.
3.1) \( Q_c = \frac{9}{16}, \frac{9}{16}, 0, 0, 0, 0 \)

3.2) \( Q_c, Q_r \) are strictly dominated. In the resulting game, following elimination of \( Q_c, Q_r \), \( Q_c, Q_r \) are strictly dominated. Thus \( (Q_c, Q_r) \) is also the only NE.

3.3) By backward induction, \( S_2^* = Q_c, Q_r, Q_c \) and \( S_1 = Q_r, Q_c \).

By backward induction, \( S_2^* = Q_c, Q_r, Q_c \) and \( S_1 = Q_r, Q_c \). This is not a subgame perfect equilibrium.

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3.5)

\[ \frac{9}{8}, \frac{9}{8}, \frac{15}{16}, \frac{3}{4}, \frac{15}{16}, 1, 1, \frac{1}{2}, \frac{9}{8}, \frac{9}{8}, \frac{3}{4}, \frac{3}{2}, 0, 0, 0, 0, \]

where \( \mu^F_c = P(2F|Sc) \), \( \mu^c_c = P(2c|Sc) \) and \( \mu^c_L = P(2c|Sc) \), and \( \mu_i^F + \mu_i^c + \mu_i^L = 1 \) \( \forall i \in \{F, C, L\} \).

3.6) \( \alpha^*_1 = Q_F \), \( \alpha^*_2 = Q_F \).

\( \mu_i = 1 \) \( \forall i \in \{F, C, L\} \) and \( \mu_j = 0 \) \( \forall i \neq j \), \( i, j \in \{F, C, L\} \)

and all \( \mu_i \) are well defined by Bayes rule.

This is a sequential eq for all \( P \in \{\frac{1}{3}, 1\} \).
4.1) \( S_1 : \{ b, g \} \rightarrow \{ a_1, n_1 \}, \quad |S_1| = 2^2 = 4 \)

\( S_2 : \{ b, g \} \times \{ a_1, n_1 \} \rightarrow \{ a_2, n_2 \}, \quad |S_2| = 2^2 = 16 \)

\( S_3 : \{ b, g \} \times \{ a_1, n_1 \} \times \{ a_2, n_2 \} \rightarrow \{ a_3, n_3 \}, \quad |S_3| = 2^8 = 256 \)

4.2) **Player 1:**

\[
M'_1 = P(G | g) = \frac{P(g|G) P(G)}{P(g|G) P(G) + P(g|B) P(B)} = \frac{\frac{1}{2}}{\frac{1}{2} + (1 - \frac{1}{2})^2} = \frac{1}{P} > \frac{1}{2}
\]

\[
M'_2 = P(G | b) = \frac{P(b|G) P(G)}{P(b|G) P(G) + P(b|B) P(B)} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2} (1 - \frac{1}{2})^2} = \frac{1}{1 - P} < \frac{1}{2}
\]

Because of the structure of payoffs, clearly each player must play \( a_i \) following \( S_i \) if \( P(G | s_i) > \frac{1}{2} \)

and \( n_i \) if \( P(G | s_i) < \frac{1}{2} \). If \( P(G | s_i) = \frac{1}{2} \), then either \( a_i \) or \( n_i \) could be supported.

\( S_i = \begin{cases} a_i, & \text{if } s_i = g \\ n_i, & \text{if } s_i = b \end{cases} \)

**Player 2:**

\[
M^2_1 = P(G | g, a_1) = \frac{P(g|G) P(a_1|G) P(G)}{P(g|G) P(a_1|G) P(G) + P(g|B) P(a_1|B) P(B)}
\]

\[= \frac{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} (1 - \frac{1}{2})^2} > \frac{1}{2} \]

\[
M^2_2 = P(G | b, n_1) = \frac{(1 - \frac{1}{2})^2}{(1 - \frac{1}{2})^2 + \frac{1}{2}^2} < \frac{1}{2} \quad \text{similarly.}
\]

\[
M^2_3 = P(G | g, n_1) = \frac{P(g|G) P(n_1|G) P(G)}{P(g|G) P(n_1|G) P(G) + P(g|B) P(n_1|B) P(B)}
\]

\[= \frac{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2} (1 - \frac{1}{2})^2 + \frac{1}{2} \cdot \frac{1}{2}} > \frac{1}{2} \]

\( M_3 = P(6 \mid b, a_1) = \frac{1}{2} \), similarly.

For this equilibrium (sequential), let's keep all histories on the path:

\[
S_2^* = \begin{cases} 
  a_2 & \text{if } S_2 = g \\
  n_2 & \text{if } S_2 = b
\end{cases}
\]

(this means that \( P(a_2, a_1 \mid g) = P(a_2 \mid g)P(a_1 \mid g) \) since player 2's strategy is independent of player 1's play. Convenient.

\[ M_3^1 = P(6 \mid g, a_1, a_2) = \frac{p_3^3}{p_2^2 + (1-p_2)^3} > \frac{1}{2} \]

\[ M_2^3 = P(6 \mid g, a_1, n_2) = M_2^3 = P(6 \mid g, n_1, a_2) = M_4^3 = P(6 \mid b, a_1, a_2) = \frac{p_3^2(1-p_2)}{p_2^2(1-p_2) + (1-p_2)^3} > \frac{1}{2} \]

\[ M_6^3 = P(6 \mid g, n_1, n_2) = M_6^3 = P(6 \mid b, a_1, n_2) = M_7^3 = P(6 \mid b, n_1, a_2) = \frac{p_2^3}{p_2^3 + (1-p_2)^3} < \frac{1}{2} \]

\[ M_8^3 = P(6 \mid b, n_1, n_2) = \frac{(1-p_2)^3}{(1-p_2)^3 + p_2^3} < \frac{1}{2} \]

\[ S_3^* = \begin{cases} 
  a_3 & \text{if } h_3 \not\in \{ (g, a_1, a_2), (g, a_1, n_2), (g, n_1, n_2), (b, a_1, a_2) \}
  n_3 & \text{if } h_3 \not\in \{ (b, n_1, n_2), (g, n_1, n_2), (b, a_1, n_2), (b, n_1, a_2) \}
\end{cases} \]

\((S^*, M)\) constitute a sequential equilibrium.

4.3) Now, we must alter player 2's strategy. In order for player 3 to adapt after \((g, n_1, n_2)\), \(M_3^3 \geq \frac{1}{2}\).

\[ S_2^* = \begin{cases} 
  a_1 & \text{if } h_2 \not\in \{ (g, a_1) \}
  n_1 & \text{if } h_2 \in \{ (g, n_1), (b, a_1), (b, n_1) \}
\end{cases} \]

Now, with this "pessimistic" choice, we can make 3 adapt.
4.3. continued

\[
M_0^2 = P(g | n, n_2 | G) P(G) \frac{P(g, n_1, n_2 | G) P(G)}{P(g, n_1, n_2 | G) P(G) + P(g, n_1, n_2 | B) P(B)}
\]

\[
P(g, n_1, n_2 | G) = P(g | G) P(n_1, n_2 | G)
\]

\[
P(n_1, n_2 | G) = \begin{aligned}
P(n_1 | g_1, g_2, g_3) & P(g_1, g_2, g_3) + P(n_1 | n_2 | g_2, b_1) P(g_2, b_1, G) \\
& + P(n_1 | n_2 | b_2, b_1) P(b_2, b_1, G)
\end{aligned}
\]

\[
= P(n_1 | n_2 | g_2, b_1) P(g_2, b_1, G) + P(n_1 | n_2 | b_2, b_1) P(b_2, b_1, G)
\]

\[
= P(1-p) + (1-p)^2.
\]

Similarly, \(P(n_1, n_2 | B) = p(1-p) + p^2.\)

Thus, plugging into \(\bigcirc\), we get:

\[
M_0^2 = \frac{1}{2}.
\]

Thus, we can structure \(S_3\) such that 3 adopts following

\((n_1, n_2, g).\)

4.4). Similar to 4.3)

4.5) we did already in 4.2.