1. Write down a proof of the revelation principle using (pure strategy) Bayesian Nash equilibrium as the equilibrium concept. That is, prove that if there exists a game form \( <M, g> \) with \( s^* \) being a pure strategy Bayesian Nash equilibrium in the Bayesian game induced by \( <M, g> \), then there exists a direct revelation mechanism \( \langle \Theta, \sigma \rangle \) in which truth-telling is a Bayesian Nash equilibrium and where the eventual outcome is the same as in the initial game for every \( \theta \in \Theta \). For simplicity you may write down the proof starting from a deterministic mechanism (in class \( g \) allowed for randomizations).

2. Carefully write down the associated direct revelation mechanisms corresponding to the following (static) auctions:
   1. A first price auction.
   2. A second price auction.
   3. An all pay auction.

3. Suppose that there is a non-truthful equilibrium in a direct revelation mechanism. Show that there is a truthful equilibrium in another direct revelation mechanism such that a truthful equilibrium that generates the same allocation as in the non-truthful equilibrium exists.

4. Suppose a seller has an object that may be worth either \( v_L = \frac{1}{4} \) or \( V_H = \frac{3}{4} \) to the seller with probability \( q \) and \( 1 - q \). A buyer has valuations \( \theta_L \) or \( \theta_H \) with \( \frac{1}{4} < \theta_L < \frac{3}{4} \) and \( \theta_H > \frac{3}{4} \) and probabilities \( p \) and \( 1 - p \).
   1. What is the efficient trading rule?
   2. Suppose that you seek a direct mechanism where:
      1. Truth-telling is incentive compatible (a Bayesian Nash).
      2. Both types of each player is better off participating than not.
      3. All transfers are between the buyer and the seller only (no outside money)

   Write down the relevant inequalities that need to be satisfied and try to determine which are the relevant ones.

3. Check for whether there are conditions when the efficient trading rule can satisfy these conditions and when it cannot.